Sir W. R. Hamilton, LL. D., read the following paper:—

On the Existence of a Symbolic and Biquadratic Equation, which is satisfied by the Symbol of Linear Operation in Quaternions.

1. In a recent communication (of June 9, 1862), I showed how the general Linear and Quaternion Function of a Quaternion could be expressed, under a standard quadrinoial form; and how that function, when so expressed, could be inverted.

2. I have since perceived, that whatever *form* be adopted, to represent the *Linear Symbol of Quaternion Operation* thus referred to, that *symbol* always satisfies a certain *Biquadratic Equation*, with *Scalar Coefficients*, of which the *values* depend upon the particular *constants* of the *Function* above referred to.

3. This result, with the properties of the *Auxiliary Linear* and *Quaternion Functions* with which it is connected, appears to me to constitute the most remarkable accession to the *Theory of Quaternions proper*, as distinguished from their *separation* into *scalar* and *vector parts*, and from their *application to Geometry* and *Physics*, which has been made since I had first the honour of addressing the Royal Irish Academy on the subject, in the year 1843.

4. The following is an outline of one of the proofs of the existence of the biquadratic equation, above referred to. Let

$$fq = r \tag{1}$$

be a given linear equation in quaternions; r being a given quaternion, q a sought one, and f the symbol of a linear or distributive operation: so that

$$f(q + q') = fq + fq', \tag{2}$$

whatever two quaternions may be denoted by q and q'.

5. I have found that the *formula of solution* of this equation (1), or the formula of *inversion* of the *function*, f, may be thus stated:

$$nq = nf^{-1}r = Fr; \tag{3}$$

where n is a *scalar constant* depending for its *value*, and F is an auxiliary and *linear symbol* of operation depending for its *form* (or rather for the *constants* which it involves), on the *particular form* of f; or on the special values of the *constants*, which enter into the composition of the *particular function*, fq.

6. We have thus, independently of the particular quaternions, q and r, the equations,

$$Ffq = nq, \ fFr = nr; \tag{4}$$

or, briefly and symbolically,

$$Ff = fF = n. \tag{5}$$

7. Changing next f to $f_c = f + c$, that is to say, proposing next to resolve the *new linear equation*,

$$f_c q = fq + cq = r, \tag{6}$$

where c is an *arbitrary scalar*, I find that the *new* formula of solution, or of inversion, may be thus written:

$$f_c F_c = n_c ; \tag{7}$$

where $$F_c = F + cG + c^2 H + c^3, \tag{8}$$

and $$n_c = n + n'c + n''c^2 + n''' c^3 + c^4 ; \tag{9}$$

G and H being the symbols (or characteristics) of *two new linear operations*, and n', n'', n''' denoting *three new scalar constants.*

8. Expanding then the symbolical product $f_c F_c$, and comparing powers of c, we arrive at *three new symbolical equations*, namely, the following:

$$fG + F = n' ; \; fH + G = n''; \; f + H = n''' ; \tag{10}$$

by elimination of the symbols, F, G, H, between which and the equation (5), the *symbolical biquadratic*,

$$0 = n - n'f + n''f^2 - n''' f^3 + f^4, \tag{A}$$

is obtained.

B. B. Stoney, B.A., read the following paper:—

On the Strength of Long Pillars.

Among the numerous difficulties encountered in designing large iron structures, such as railway girders or roofs of large span, none perhaps is of more importance, or requires greater skill to overcome, than the tendency of parts under compression to deflect beneath the pressure, and yield sideways, like a thin walking-cane, when the load is greater than it can support without bending.

To understand the matter clearly, we must recollect that the mode in which a pillar fails varies greatly, according as it is long or short in proportion to the diameter. A very short pillar—a cube, for instance—will bear a weight sufficient to splinter or crush it into powder; while a still shorter pillar—such as a penny, or other thin plate of metal—will bear an enormous weight, far exceeding that which the cube will sustain, the interior of the thin plate being prevented from escaping from beneath the pressure by the surrounding particles. We can thus conceive how stone or other materials in the centre of the globe withstand pressures that would crush them into powder at the surface, merely because there is no room for the particles to escape from the surrounding pressure.

It has been found by experiment that the strength of short pillars of any given material, all having the same diameter, does not vary much, provided the length of the pillar is not less than one, and does not exceed four or five diameters; and the weight which will just crush a short pillar, one square inch in section, and whose length is not less than one or greater than five inches, is called the *crushing strength* of